DEPARTMENT OF MECHANICAL ENGINEERING
College of Engineering Thalassery

ME202 Advanced Mechanics of Solids

Tutorial-3: Stress-strain relations and 2D problems in elasticity

1. Using the stress strain relations, strain compatibility relations, and equations of equilibrium, derive the relationship for plain strain $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\sigma_x+\sigma_y) = \frac{-1}{1-\nu}\left(\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}\right)$ X and Y are body force components along x and y directions respectively and ν is the Poisson's ratio.

2. For steel, the following data is applicable: $E = 206\times10^6 kPa, \quad G = 80\times10^6 kPa$. For the given strain matrix at a point, determine the stress matrix.

$$[\epsilon_{ij}] = \begin{bmatrix} 0.001 & 0 & -0.002 \\ 0 & -0.003 & 0.003 \\ -0.002 & 0.003 & 0 \end{bmatrix}$$

3. A thin rubber sheet is enclosed between two fixed hard steel plates (see Fig.1). Friction between the rubber and steel faces is negligible. If the rubber plate is subjected to stresses σ_x and σ_y as shown, determine the strains ϵ_x and ϵ_y and also the stress σ_z.

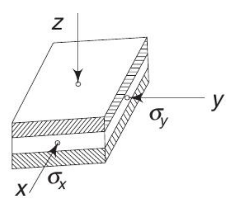

Figure 1: Problem-3

4. Investigate what problem solved by $\phi = \frac{F}{d^3}xy^2(3d-2y)$ applied to the region included in $y=0, y=d, x=0$ on the side x positive.

5. Determine whether the following $\phi = \frac{3F}{4h}\left(xy-\frac{xy^3}{3h^2})\right)+\frac{P}{2}y^2$ can be used as a stress function.If so, determine the components of stress represented by it.

6. Obtain the expressions for σ_x and σ_y in terms of Lame's constants, strain components and dilation.

7. Show that $\phi = A\left(xy^3-\frac{3}{4}xyh^2\right)$ is an Airy's stress function, where A and h are constants. Also show that it represents the stress distribution in a cantilever beam loaded at free end with a point load. Find the value of A if b and h are width and depth respectively.

8. The state of stress at a point is given by $[\sigma_{ij}] = \begin{bmatrix} 9 & 0 & 3 \\ 0 & -10 & 1 \\ 3 & 1 & 112 \end{bmatrix}\times10^3 kPa$ compute the strain tensor for the material with $E = 206\times10^6 kPa, \quad and \quad G = 80\times10^6 kPa$

9. Prove that $\phi = Ax^3$ represents Airy's stress function of a 2D problem. Find the stress components.

10. Obtain a polynomial stress function such that σ_y varies linearly in the x direction only. All the other components of stress are equal to zero.

11. Derive the stress compatibility equation of a 2D plane stress elasticity problem.

12. Derive the stress compatibility equation in terms of Airy's stress function for 2D plane stress problem (i) If body force is absent (ii)If body force is present.

13. Derive the stress compatibility equation in terms of Airy's stress function for 2D plane strain problem (i) If body force is absent (ii)If body force is present.

14. Derive the bending and shear stress distribution in a cantilever loaded at its free end.