DEPARTMENT OF MECHANICAL ENGINEERING
College of Engineering Thalassery

ME202 Advanced Mechanics of Solids

Tutorial-2: Analysis of Strain

1. The general displacement field in a body, in Cartesian co-ordinates, is given by $u = 0.015x^2y + 0.03, v = 0.005y^2 + 0.03xz, w = 0.003z^2 + 0.001yz + 0.005$. Find the strain tensor, ϵ_{ij} for point (1,0,2)

2. The displacement field in a body is given by $[(x^2 + y^2 + 2)\mathbf{i} + (3x + 4y^2)\mathbf{j} + (2x^3 + 4z)\mathbf{k}] \times 10^{-4}$. Obtain the magnitude of principal strains at location (1,2,3)

3. Verify whether the following strain field satisfies equations of compatibility $\epsilon_x = Py, \quad \epsilon_y = Px, \quad \epsilon_z = zP(x + y), \quad \epsilon_{xy} = P(x + y), \quad \epsilon_{yz} = 2Pz \quad and \quad \epsilon_{zx} = 2Pz$. Where P is a constant.

4. If the displacement field is given by $u_x = Kxy + y, u_y = Kxy, u_z = 2K(x + y)$, where K is a constant.
   (a) Write down the strain matrix.
   (b) What is the strain in direction $n_x = n_y = n_z = 1/\sqrt{3}$

5. The displacement field for a body is given by $\mathbf{u} = [(x^2 + y^2 + 2)\mathbf{i} + (3x + 4y^2)\mathbf{j} + (2x^3 + 4z)\mathbf{k}]$What is the displaced position of a point originally at (1, 2, 3)?

6. The displacement field is given by $u_x = k(x^2 + 2z), u_y = k(4x + 2y^2 + z), u_z = 4kz^2$ k is a very small constant. What are the strains at (2, 2, 3) in directions
   (a) $n_x = 0, n_y = 1/\sqrt{2}, n_z = 1/\sqrt{2}$
   (b) $n_x = 1, n_y = n_z = 0$
   (c) $n_x = 0.6, n_y = 0, n_z = 0.8$

7. The rectangular components of a small strain at a point is given by the following matrix. Determine the principal strains and the direction of the maximum linear strain

$$[\epsilon_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -4 \\ 0 & -4 & 3 \end{bmatrix}$$

8. For the displacement field given by $u_x = K(x^2 + 2z), u_y = K(4x + 2y^2 + z), u_z = 4Kz^2$, where K is a very small constant, verify whether the compatibility conditions are satisfied.

9. State the conditions under which the following is a possible system of strains:

$$\epsilon_{xx} = a + b(x^2 + y^2)x^4 + y^4, \gamma_{yz} = 0$$

$$\epsilon_{yy} = \alpha + \beta(x^2 + y^2) + x^4 + y^4, \gamma_{xz} = 0$$

$$\gamma_{xy} = A + Bxy(x^2 + y^2 - c^2), \epsilon_{zz} = 0$$

10. The tensorial strain at the origin is given by

$$[\epsilon_{ij}] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & 3 & 1 \end{bmatrix} \times 10^{-2}$$

    Determine normal strain along the direction from (0,0,0) to (3,4,12)