DEPARTMENT OF MECHANICAL ENGINEERING
College of Engineering Thalassery

ME202 Advanced Mechanics of Solids

Tutorial-1: Analysis of Stress

1. A rectangular beam is subjected to a pure bending moment M. The cross section of the beam is shown in Fig. 1. Using the elementary flexure formula, determine the normal and shearing stresses at a point (x, y) on the plane AB shown.

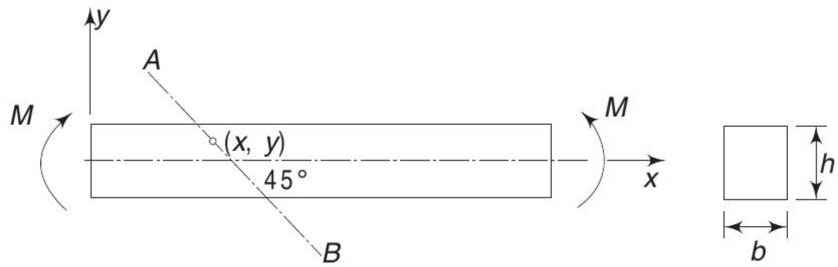

Figure 1: Problem 1

2. The state of stress at a point is characterised by the matrix shown. Determine T_{11} such that there is at least one plane passing through the point in such a way that the resultant stress on that plane is zero. Determine the direction cosines of the normal to that plane.

$$\tau_{ij} = \begin{bmatrix} T_{11} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

3. If the rectangular components of stress at a point are as in the matrix below, determine the unit normal of a plane parallel to the z axis,i.e. $n_z = 0$, on which the resultant stress vector is tangential to the plane

$$\tau_{ij} = \begin{bmatrix} a & 0 & d \\ 0 & b & e \\ d & e & c \end{bmatrix}$$

4. Determine the principal stresses, their axes, principal shears and the associated normal stresses for the states of stress characterised by the following stress matrices (units are 1000 kPa).

(a)

$$\tau_{ij} = \begin{bmatrix} 18 & 0 & 24 \\ 0 & -50 & 0 \\ 24 & 0 & 32 \end{bmatrix}$$

(b)

$$\tau_{ij} = \begin{bmatrix} 3 & -10 & 0 \\ -10 & 0 & 30 \\ 0 & 30 & -27 \end{bmatrix}$$

(c)

$$\tau_{ij} = \begin{bmatrix} 12.31 & 4.20 & 0 \\ 4.20 & 8.96 & 5.27 \\ 0 & 5.27 & 4.34 \end{bmatrix}$$

5. A solid shaft of diameter $d = \sqrt{10}cm$ is subjected to a tensile force $P = 10,000N$ and a torque$T = 5000Ncm$. At a point on the surface, determine the principal stresses, the octahedral shearing stress and the maximum shearing stress.

6. The state of stress at a point for a given reference is given below. If a new set of axes $x'y'z'$ is formed by rotating xyz through 60^Oabout z axis,find the new stress tensor

$$\tau_{ij} = \begin{bmatrix} 200 & 100 & 0 \\ 100 & 0 & 0 \\ 0 & 0 & 500 \end{bmatrix}$$

7. At a point P, the rectangular stress components are, $\sigma_x = 1, \sigma_y = -2, \sigma_z = 4, \tau_{xy} = 2, \tau_{yz} = -3$ and $\tau_{zx} = 1$ all in units of KPa. Find the principal stresses and directions.

8. Find the principal stresses and check for invariance in the following case.

$$\tau_{ij} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -2 & -3 \\ 1 & -3 & 4 \end{bmatrix}$$

9. The stress at a point is given by the three principal stresses 100, 120 and $200N/mm^2$. Determine the shear and normal stresses on a plane which has normal with direction cosines as $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ and 0.

10. The state of stress at a point is characterized by the components $\sigma_x = 100, \sigma_y = -40, \sigma_z = 80, \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$ all in $10^6 N/m^2$. Determine the extreme values of shear stresses, their associated normal stresses, the octahedral shear stress and its associated normal stress.

11. The state of stress at a point is given by $\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = \rho$. Determine principal stresses and directions.

12. A cross-section of the wall of a dam is shown in Figure 2. The pressure of water on face OB is also shown. The stresses at any point (x, y) are given by the following expressions

$$\sigma_x = -\gamma y$$

$$\sigma_y = \left(\frac{\rho}{tan\beta} - \frac{2\gamma}{\tan^3 \beta} \right)$$

$$\tau_{xy} = \frac{-\gamma x}{\tan^2 \beta}$$

$$\tau_{yz} = \tau_{xz} = \sigma_z = 0$$

where γ is the specific weight of water and ρ the specific weight of the dam material.
Consider an element OCD and show that this element is in equilibrium under the action of the external

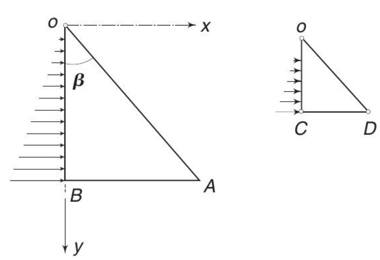

Figure 2: Problem 12

forces (water pressure and gravity force) and the internally distributed forces across the section CD

13. At a point in a stressed material, the principal stresses acting are given by, $\sigma_1 = 120Pa, \sigma_2 = 60Pa, \sigma_3 = 20Pa$. Find the normal and shear stress on a plane whose normal is inclined at an angle of 40^O to the σ_1 axis in the plane containing σ_1 and σ_3 stresses and 50^O to the σ_1 axis in the plane containing σ_1 and σ_2 stresses. Find also the normal and shear stresses on Octahedral Planes.

14. The state of stress at a particular point relative to the xyz coordinate system is given by the stress matrix

$$\tau_{ij} = \begin{bmatrix} 15 & 10 & -10 \\ 10 & 10 & 0 \\ -10 & 0 & 40 \end{bmatrix} MPa$$

Determine the normal stress and magnitude and direction of the shear stress on a surface intersecting the point and parallel to the plane given by the equation $2x - y + 3z = 9$

15. (a) Decompose the given stress in to hydrostatic and deviatoric parts.

$$\tau_{ij} = \begin{bmatrix} 57 & 0 & 24 \\ 0 & 50 & 0 \\ 24 & 0 & 42 \end{bmatrix} kPa$$

(b) What is the octahedral normal and shear stress of hydrostatic part

(c) What is the octahedral normal and shear stress of deviatoric part?

16. The stress field of a body is given by $\sigma_x = 20x^2 + y^2$, $\sigma_y = 30x^3 + 200$, $\sigma_z = 30(y^2 + z^2)$, $\tau_{xy} = zx$, $\tau_{xz} = y^2 z$, $\tau_{yz} = x^3 y$. Find out the components of body force required for satisfying the equilibrium of the body